COMBINING VERTICAL ACCELERATION DATA WITH GPS AND VERTICAL VELOCITY DATA

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Abstract

Many independent studies very different in nature point towards the same conclusion: that the wing of the TU-154 that crashed in Smolensk in 2010 was not cut by a birch tree. Black box data and studies clearly suggest that the plane instead was more than 28m and most likely 69 m above the ground of the birch tree officially claimed to have cut the wing and 30m north of this. Nevertheless the wing tip was separated from the rest of the plane at a distance of about 460m prior to the crash site. The data clearly point toward the plane losing its wing tip in free air space where no obstacles were present.

A TU-154M plane or similar losing its wing unmotivated in free air space has never earlier been reported to happen on any of the commercial airplanes including all the thousand of built TU-154 planes flying many million miles in all types of rough weather around the globe. This points toward a provoked rather than unmotivated separation.

In theory an integration of the black box recorded vertical acceleration data should lead to knowledge of the change in the planes velocity, and another integration of these velocity data should lead to knowledge of the change in the planes height. In practice however it is well known, that the results hereof will be strongly influenced by any existing signal error such as a simple scale or bias error or an error in the signal caused by an average instrument angle etc.

The work presented here utilizes the height changes as measured by the three GPS units and recorded at the TAWS 35 to TAWS 37 events together with the logged vertical speeds at this points to reduce the effect of the various sources of error on the vertical acceleration sensor data, allowing for an accurate determination of the planes height through a simple double integration.

Using this mathematical technique the calculated position (X,Y and Z) of the plane when it lost its left wing tip agrees within a few meters with the position calculated through independent data based on knowledge of the aerodynamic performance of the damaged plane working backwards from the crash site and up as presented at the Smolensk Conference 2014 in Warsaw. In the latter the aerodynamic data are obtained through state of the art CFD calculations done by Metacomp Inc. USA, one of the world's leading companies within this field and a sub supplier of Boeing.

The effect of the measurement uncertainties is investigated using a Monte Carlo technique showing the plane with 99.9% certainty flew more than 28m above the ground in the vicinity of the Birch Tree claimed by the Russians to have cut the wing 5m above the ground.

Keywords *- GPS data, wing damage, roll, Smolensk, TU-154, Monte Carlo technique.*

Streszczenie

Wiele niezależnych badań o całkiem różnej naturze prowadzi do tego samego wniosku, że skrzydło samolotu TU-154, który rozbił się w Smoleńsku w 2010 roku, nie zostało obcięte przez drzewo brzozy. Dane z czarnych skrzynek i analizy wyraźnie sugerują, że samolot był na wysokości większej niż 28 m - a najbardziej prawdopodobnie 69 m - powyżej powierzchni ziemi w miejscu brzozy oficjalnie obwinianej obcięcie skrzydła i 30 m na północ od niej. Niemniej jednak końcówka skrzydła została oddzielona od reszty samolotu w odległości około 460 m przed miejscem uderzenia w ziemię. Dane wskazują, że samolot stracił końcówkę skrzydła w wolnej przestrzeni, gdzie nie istnieją żadne przeszkody.

Nigdy wcześniej nie odnotowano by samolot TU-154 lub podobny stracił skrzydło bez powodu w wolnej przestrzeni powietrznej. Nie zdarzyło się to jakiemukolwiek handlowemu samolotowi włączając w to cały tysiąc zbudowanych samolotów TU-154, które przeleciały wiele milionów mil we wszelkich surowej warunkach pogodowych na całym globie. Wskazuje to na raczej sprowokowane, a nie bezprzyczynowe oddzielenie.

Całkowanie danych zarejestrowanego w czarnej skrzynce przyspieszenia pionowego powinno teoretycznie dawać zmiany szybkości samolotu, a całkowanie danych dotyczących tej prędkości powinno dawać zmiany wysokości samolotu. W praktyce jednak jest dobrze wiadome, że wyniki będą silnie zależeć od istniejących błędów sygnału takich jak błędy skali lub pochylenia, albo błąd sygnału spowodowany przez średni kąt instrumentu itp.

Przedstawiona praca wykorzystuje zmiany wysokości zmierzone przez 3 jednostki GPS i zarejestrowane od wydarzenia TAWS 35 do TAWS 37 razem z wpisanymi pionowymi prędkościami w tych punktach, by zredukować efekt różnych źródeł błędu na dane czujnika przyspieszenia pionowego pozwalając na dokładne ustalenie wysokości samolotu przez proste podwójne całkowanie.

Obliczania przy tej matematycznej technikę pozycja (X, Y i Z) samolotu, kiedy utracił on swą końcówkę lewego skrzydła, zgadza się z dokładnością do kilku metrów z pozycją obliczoną przez niezależne dane oparte na analizie aerodynamicznych zachowań uszkodzonego samolotu prowadząc obliczenia wstecz od miejsca katastrofy w górę, jak to przedstawiono na Konferencji Smoleńskiej w Warszawie w 2014 roku. W tych drugich obliczeniach dane aerodynamiczne zostały uzyskane przez zastosowanie obliczeń CFD przeprowadzonych przez Metacomp Inc. w USA, jedną z wiodących firm w tej dziedzinie, współpracującej z firmą Boeing.

Efekt niepewności pomiarowych został zbadany przy użyciu metody Monte Carlo pokazując, że samolot z prawdopodobieństwem 99,9 % leciał wyżej niż 28 m nad gruntem w okolicy brzozy oskarżanej przez Rosjan, że obcięła skrzydło 5 m ponad gruntem.

Słowa kluczowe *– dane GPS, uszkodzenie skrzydła, beczka, Smoleńsk, TU-154, metoda Monte Carlo.*

1. INTRODUCTION

The TU-154M plane that crashed in Smolensk on the 10th of April 2010 had in total 5 black boxes on board. One of these containing valuable data was officially never found despite this unit was the one mechanically protected best on board. Just a week after the crash the Russians released the TAWS black box and the two FMS boards to the American

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company Universal Avionics. The data of these units are now believed to be in the hands of the American NTSB.

Based on the specific list of parameters ordered by the Russian and Polish authorities a selected small subset of the data available in these units - and these selected parameters only - were retracted from the black box and FMS boards and published in two reports [1, 2].

No reason is given to why the investigation team did not request the full amount of data, when they easily could have done so. Strong indications exist, that the remaining data contain valuable information that can enlighten the investigation and further question the official investigation, and the author strongly encourages the Polish authorities to obtain and publish the full amount of the data. Examples hereof are the amount and nature of the errors that occurred in air before the crash, and the full amount of GPS data including the measured GPS height at the point where the FMS recorded its power loss (in air).

The work presented here utilizes the small subset of data selected by the Russian and Polish authorities and published by the American company Universal Avionics. In particular the GPS heights and vertical speeds recorded at the TAWS 34 to 38 events are of interest in this work. The information of these data is combined with the vertical acceleration sensor data recorded by the Polish QAR data recorder on board.

In theory an integration of the black box recorded vertical acceleration data should lead to knowledge of the change in the planes velocity, and another integration of these velocity data should lead to knowledge of the change in the planes height. In practice however it is well known, that the results hereof will be strongly influenced by any existing signal error such as a simple scale and or bias error or an error in the signal caused by an instrument angle etc.

In a previous reported study based on non-calibrated vertical acceleration data only correcting for roll and pitch angle effects the calculated heights and vertical velocities do not correlate very well with the recorded values [3]. For instance the vertical velocity near the Taws 38 event is in the mentioned study found as +12.4 m/s m/s much different from the recorded value of 394 ft/min or +2 m/s.

The work presented here utilizes the height changes as measured by the three GPS units and recorded at the TAWS 35 to TAWS 37 events together with the logged vertical speeds at these points to reduce the effect of the various sources of error on the vertical acceleration sensor data (including effects of average roll and pitch), allowing for an accurate determination of the planes height through a simple double integration.

2. MODEL

The mathematical model used to combine the GPS data, vertical velocity data and vertical acceleration data is very simple. During the final descend of the plane the TAWS system recorded a series of events named TAWS 34, TAWS 35, TAWS 36, TAWS 37 and TAWS 38. At each event certain data were logged. Of interest here are the GPS positions and heights and the vertical velocities (see Tab. 1).

Normally the linearity of an acceleration sensor is very good, and the majority of any calibration error can be described as a first order function through a slope coefficient, "*a*", and a bias value, "*b*". The true vertical normalized acceleration as a function of time (*t*), $A^n_{TRUE}(t)$,

Tab. 1. The input data [1, 2]. H_{26} is the height above runway 26 assuming runway 26 has an altitiude of 255 m (MSL). Data at TAWS 35* are interpolated between TAWS 35 and TAWS 36. TAWS 38 is not included in the model but used as a control point to check the final results.

TAWS	TAWS	GPS	GPS	Sink	Sink
#	Time	H_{MSL}	H_{26}	*Vz*	*Vz*
[-]	[hr:m:s]	[ft]	[m]	[ft/min]	[m/s]
34	06:40:03	2132	394.9	-1441	-7.32
35	06:40:29	1595	231.3	-1336	-6.79
35*	06:40:30	1565	222.0	-1364	-6.93
36	06:40:36	1410	174.0	-1513	-7.69
37	06:40:43	1264	130.4	-1505	-7.65
38	06:40:59	1002	50.5	+394	+2.00

can then be related to the measured normalized vertical acceleration, $A^n_{MEASURED}(t)$, by

$$A^n_{TRUE}(t,a,b) = a * A^n_{MEASURED}(t) - b\,. \qquad (1)$$

A perfect calibrated sensor would have $a = 1$ and $b = 0$, and for a sensor of this type the "*a*" coefficient is expected to be within this by a few percent ($1.02 \geq a \geq 0.98$). The physical upwards acceleration is found by

$$A_{TRUE}(t,a,b) = (A^n_{TRUE}(t,a,b) - 1) * g\,, \qquad (2)$$

where g is the gravitational constant g = 9.81 m/s^2. Thus $A^n_{TRUE} = 1$ for a plane flying horizontal will result in the acceleration of $A_{TRUE} = 0$ m/s^2.

If (*a*,*b*) of the particular sensor are known the integration of A_{TRUE} will provide information of the true vertical velocity, V_{TRUE}, as a function of time, *t*, by:

$$V_{TRUE}(t,a,b) = \int_{t_0}^{t} (A_{TRUE}(t,a,b) * dt + V_0 \qquad (3)$$

Where V_0 is the vertical velocity at the time $t = t_0$ at beginning of the integration,. Integrating one more time provides information of the true height, H_{TRUE}, as a function of time by:

$$H_{TRUE}(t,a,b) = \int_{t_0}^{t} V_{TRUE}(t,a,b) * dt + H_0 \qquad (4)$$

Where H_0 is the height at the time $t = t_0$ at beginning of the integration.

The embedded algorithms behind the calculation of the GPS positions (*X, Y, Z*) typically improve accuracy as they track along the moved path specially when the path is near to straight lines (as the case is for an airplane with a big mass like the TU-154M). Minimum 4 satellites are required to perform a measurement, and the more satellites the GPS units can read the better the accuracy normally will be as the GPS system utilizes the redundancy to eliminate outliers and improve the estimate of the whole number of code lengths between the antenna and the given satellite. By the recorded data [2] there is reason to believe the GPS units could read about 13 satellite signals and utilized about 11 of these when approaching Smolensk. Typically during descend of a big airplane the GPS accuracy is very good, as the GPS units have been working and improving their accuracy for some duration while flying a straight line earlier under ideal

conditions with direct lines of view between the GPS antennas and the satellites and due to the fact that ground reflections of the satellite signals are strongly dampened (compared to when the GPS unit is close to the ground). Due to the geometry the vertical accuracy is typically slightly lower than the horizontal accuracy.

Even though there on this behalf are very good reasons to assume the GPS accuracy of the height determination at each TAWS event during the initial descend was good (say even better than +/-15 m) this is not super critical for the method of determining (a,b) used in this work. The reasons for this are, that i) the method utilizes all three individual measurements, ii) the method takes advantage of the height changes and iii) the absolute height change over the integrated distance is large compared to the inaccuracy of each point. This is investigated further in the following chapter describing the Monte Carlo technique.

Assuming a set of (a,b) values one can by integration find the corresponding heights, H_{35*}, H_{36}, H_{37} and velocities V_{35*}, V_{36} and V_{37} at the TAWS events and also the average velocity $V_{avg,calc}$ from TAWS 35* to TAWS 37 by equations (1), (2) and (3). (35* denotes the position close to TAWS 35 where the provided ATM data begin. Data at the 35* position are found as a linear interpolation between TAWS 35 and TAWS 36, which due to the planes large mass and thereby inertia is reasonable to do.)

The goal is to obtain the set of (a,b) that result in the best agreement between the recorded GPS data, the recorded vertical speed data and the vertical acceleration data. This is done by minimizing the squared error defined by:

$$E_V(a,b) = \sum_{35*to37} (V_{meas} - V_{calc}(a,b))^2 \qquad (5)$$

Where V_0 is found such that the average measured velocity from TAWS 35* to TAWS 37 , $V_{avg,\,meas}$, equals the calculated average velocity.

$$V_{avg,meas} = V_{avg,calc} \qquad (6)$$

A typical E_V curve is shown in Fig. 1.

H_0 is found by minimizing the squared error defined by:

$$E_H(a,b) = \sum_{35*to37} (H_{meas} - H_{calc}(a,b))^2 \qquad (7)$$

Using the same set of (a,b) found through minimizing E_v.

A typical E_H curve is shown in Fig. 2.

The number of data points are of course to few to determine both the "a" and "b" values, but for a given "a" value the best corresponding "b" value can be found that will result in the minimum E_v by (5).

3. RESULTS

The scale factor and sensor bias will in real life most likely be non-ideal, i.e. differ from a=1 and b=0. The effect of such errors are studied in the following by assuming even relative large scale factor errors, and the resulting influence on the calculated trajectory is minimal.

By doing a parametric study, it turns out, that within a large span of "a" values say even as large as ±10% (1.10 ≥ a ≥ 0.90) the resulting trajectories are practically the same for all the "a" values, when the corresponding b value is found by minimizing E_v.

The calculated trajectories for the large span in scale factors from a = 0.9 to a = 1.1 are shown in Fig. 3 and values are found in Tab. 2. The trajectories are practically identical for the investigated wide range of scale factors going from 0.9 to 1.1 and the height of the plane at the moment it lost its wing tip can be found as 55 m ± 5 m for the entire investigated scale factor span (see Fig. 3).

The conclusion of the scale and bias analysis is, that even relative large scale factor errors will lead to nearly the same results by the described method, i.e. the method is robust in calibrating the vertical acceleration sensor

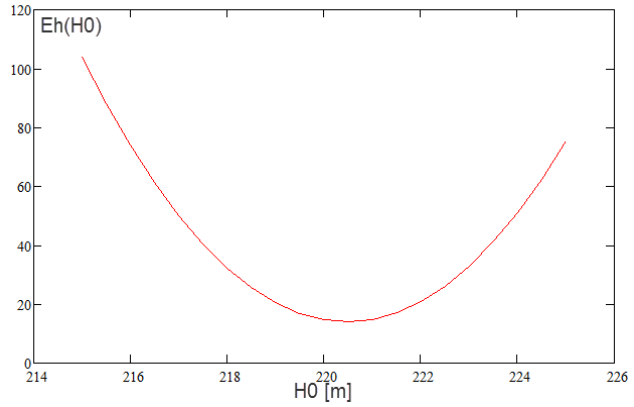

Fig. 1. A typical curve pattern for the least squared error sum Eh(H0) as a function of the initial height H0 for a given scale factor and bias (here a = 1.0, b = 1.035). In this case the best fit is obtained with H0 = 220.5 m.

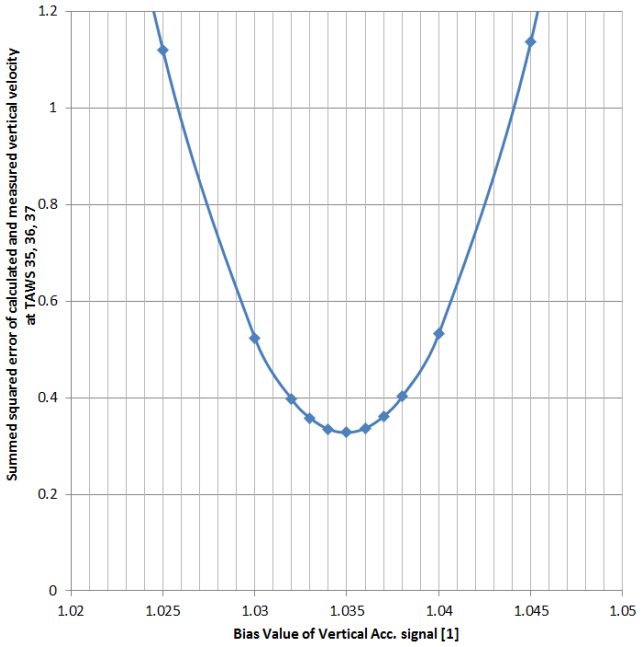

Fig. 2. A typical curve pattern for the least squared error sum EV(b) as a function of the bias for a given scale factor (here a = 1.0). In this case the best fit is obtained with b = 1.035

3.1. Monte Carlo Simulation.

The effect of measurement uncertainties of all the input parameters is investigated using a Monte Carlo Simulation with the uncertainty estimates of each input parameter as listed in Tab. 3 and N = 100.000 simulations finding the best least squared error fits as described above for each of these simulations. The results are shown in Fig. 4 and Fig. 5.

The most likely height above runway of the plane at the time the plane flew in the vicinity of the Bodin Birch tree is H_{rwy} = 57 m and the plane is with 99.9 % certainty 28 m above the ground near the birch tree (see Fig. 4 and Fig. 5).

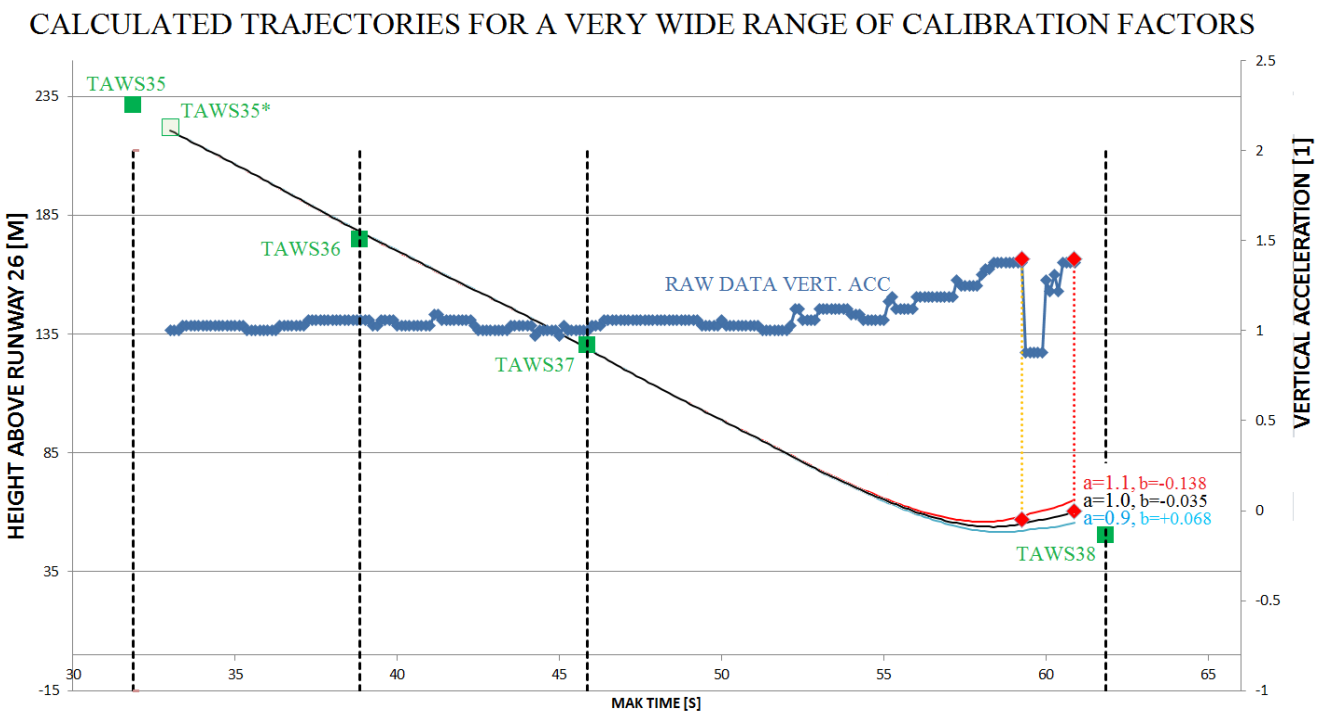

Fig. 3. The calculated trajectories based on calibrated vertical acceleration data. The calibration factors (scale factor "a" and bias "b") are found such the best agreement between the measured GPS positions, the measured vertical velocities and the recorded vertical acceleration data is obtained (minimizing the squared error sums).

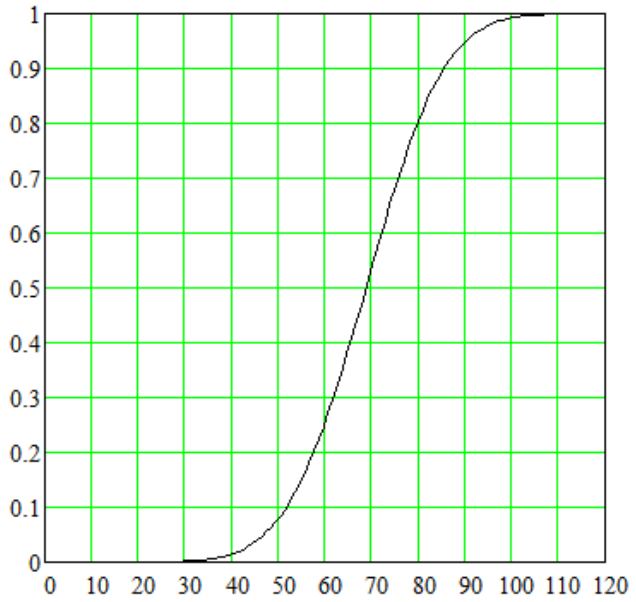

Fig. 4. The cumulative probability distribution of height (*H*) above ground in the vicinity of the Bodin Birch Tree for the *N* =100.000 Monte Carlo simulations. The plane is with *P* = 99.9 % certainty higher than 28 m above the ground near the birch tree and most likely *H* = 69 m ± 9 m above the ground according to average and median of the Monte Carlo Simulations.

Additional Information

By the Russian Final Report [4]: The elevation of the site of impact on the birch tree is 248 m (MSL) (See [4], page 76), the elevation of the runway is 255 m (MSL) (See [4], page 58) and the site of impact was 5 m above the ground of the birch tree (see [4], page 74). Thus the ground of the birch tree is ΔH = (255 m – 248 m + 5 m) = 12 m lower than the runway. This is also shown in the Final Russian Report from the Investigation Team in figure 46 page 157 of [4].

Tab. 2. Values found for three different scale factors. All parameters are defined and found through eq. (1) to eq. (7).

Scale	Bias					
a [1]	b [1]	V_0 [m/s]	H_0 [m]	H_{end} [m]	E_v [1]	E_H [1]
1.10	-0.1375	-7.02	220.7	66.4	0.330	11.8
1.00	-0.0350	-6.98	220.6	60.5	0.328	13.5
0.90	+0.0680	-6.96	220.6	55.6	0.329	14.6

Tab. 3. The estimated uncertainty values and type of uncertainty distributions used in the Monte Carlo Simulation.

Parameter of Interest	Estimated Uncertainty	Type of Distribution
GPS Height	$D_{3\sigma}$ = 30 m	Gaussian
Vertical Speed	$H_{2\sigma}$ = ±35 ft/min	Gaussian
Time of Taws Event	ΔT = ±0.5 s	Uniform
Each Individual Vertical Acc. Data Point	σ = 0.01g	Gaussian
Scale Factor	σ = 0.01	Gaussian

The 100.000 Monte Carlo simulations taking uncertainties on the input parameters into account show:

1) the plane with 99.9% certainty flew more than 28m above the ground near the Bodin Birch tree and
2) the plane most likely flew 69m above the ground near the Bodin Birch tree claimed by the Russians to have cut the wing. (This equals H_{rwy}=57m above the runway ground.).

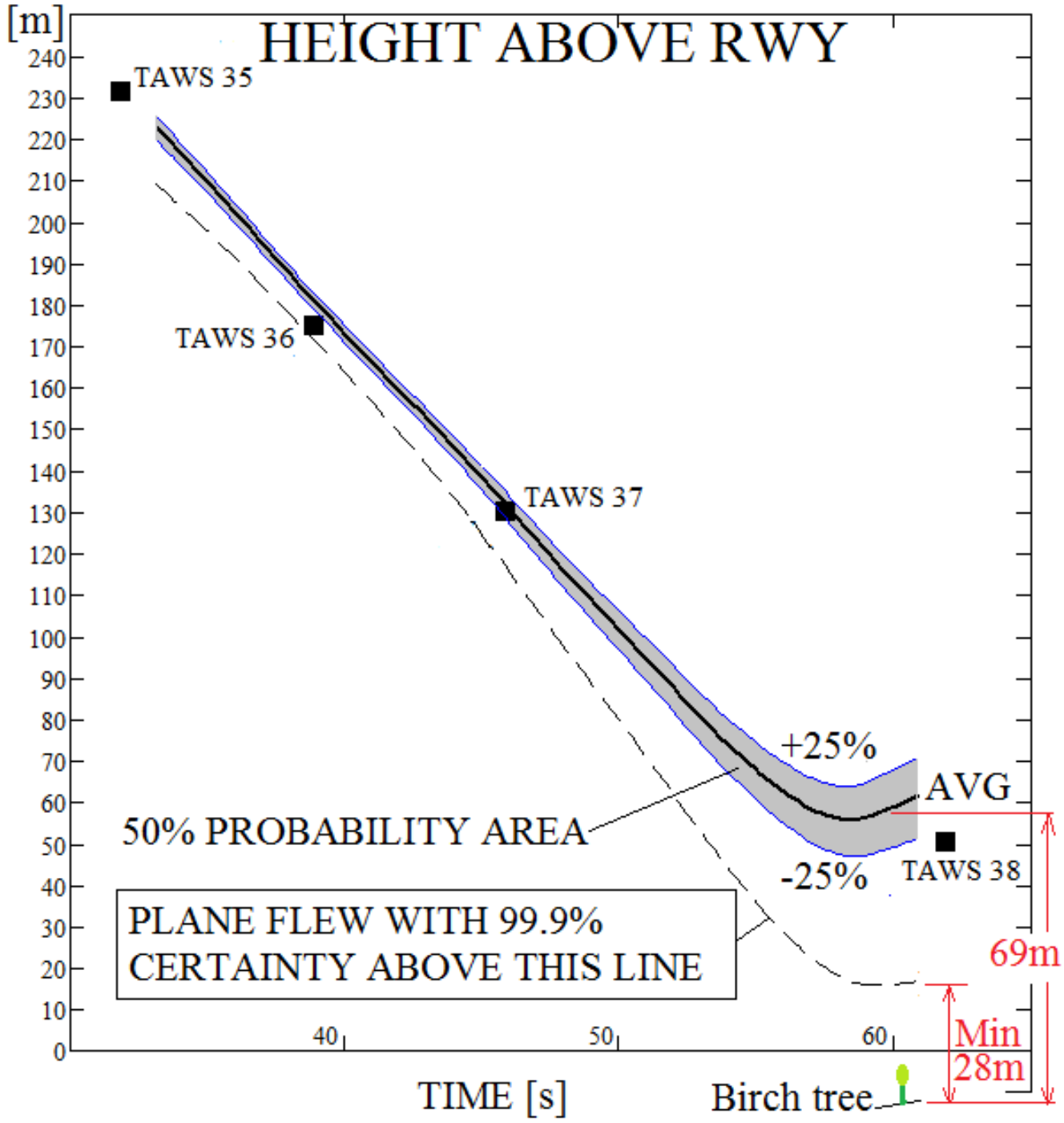

Fig. 5. Based on the recorded black box GPS heights, vertical velocities, times of logging and N = 100.000 Monte Carlo simulations the plane is with 99.9 % certainty 28 m above the ground at the vicinity of the Bodin Birch Tree, and most likely *H* = 69 m ± 9 m above this (Black Line). (The ground of the Birch Tree is 12 m below the level of runway 26 [4]).

3.2. Vertical Velocity.

The calculated vertical velocity is found for the most likely trajectory. The result is presented in Fig. 6, and this shows a very good agreement between the calculated and measured vertical velocities even at the Taws 38 event. This confirms the method as TAWS 38 is not included in the least squared error fit, but an independent data point. The 2-3 m/s lower vertical velocity at this point can be explained as a result of the additional wing loss occurring just prior to this (at the last provided vertical acceleration data point) as described in [6].

When including the effect of the additional wing loss the predicted velocity at TAWS 38 agrees with the recorded velocity at TAWS 38 for the most likely trajectory, hereby confirming the model.

3.3. Comparison to Previous Results.

The most likely trajectory found here agrees completely with the independently found trajectory based on knowledge of the aerodynamic performance of the damaged plane working backwards from the crash site and up as shown in Fig. 7. The aerodynamic data [5] are obtained through state of the art CFD calculations done by Metacomp Inc. USA,

one of the world's leading companies within this field and a sub supplier of Boeing. The bottom up trajectory based on aero dynamics was presented at the Smolensk Conference 2014 [6].

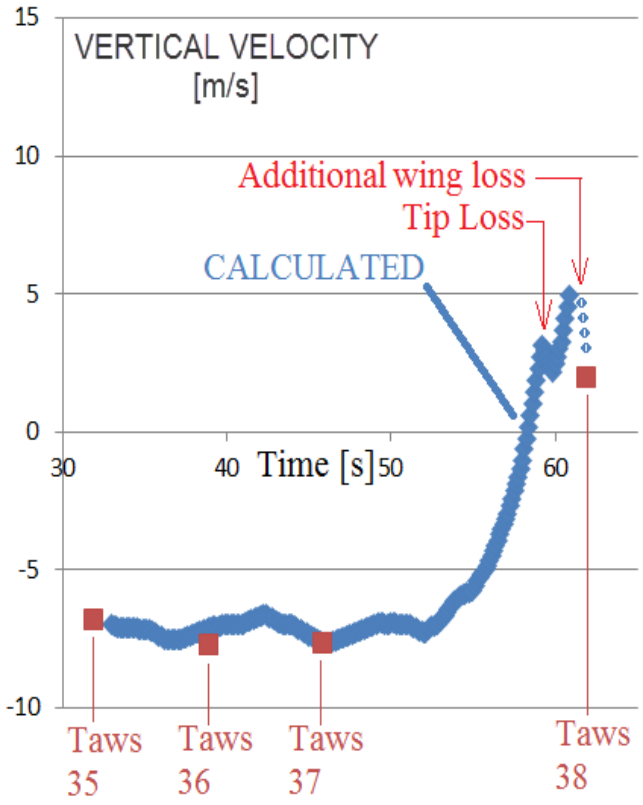

Fig. 6. The calculated vertical velocities (blue points) and the recorded vertical velocities (red squares) at the taws 35-38 events for the most likely trajectory (see average curve of figure 4).

Two totally different methods based on two independent sets of data both give the same result: The height of the plane was about 55 m above the runway altitude, when it lost the first part of its left wing.

NOTE : No obstacles exist at this height.

The calculated height loss during the go-around agrees well with the expected height loss of the TU-154M as by the Russian litterature confirming the results.

3.4. Additional Results

From Fig. 8 it can be seen, the plane was at 100 m height above the runway, at the time the pilots according to the official Russian report announced they would abort the landing procedure and initiate the go-around (see red circle of Fig. 8). The data suggest, the pilots initiated the go-around within the second after their announcement on the radio. The calculated height loss (32 m – 46 m) is in good agreement with the expected value for a TU-154M plane with the downwards vertical speed of about 6.95m/s at the moment the Go-Around was initiated (see Fig. 9 and Fig. 10). Note the trajectory by the official report does not agree with the recorded GPS measurements. The radio heights and navigator's readings do support the official Russian trajectory, but these are relative simple to manipulate (opposite the GPS recordings) and are by the authors opinion both most likely manipulated explaining why the Baro height at TAWS 38 is inconsistent with those of TAWS 34 to TAWS 37. The blue data of fig. 8 show the raw data of the vertical acceleration sensor (ATM).

The data show the pilots initiated the go-around within the second after they announced they would do this. This is in full agreement with what is normally expected from competent pilots, namely that they actually follow the command they loudly call in the cockpit, and as such not at all surprising.

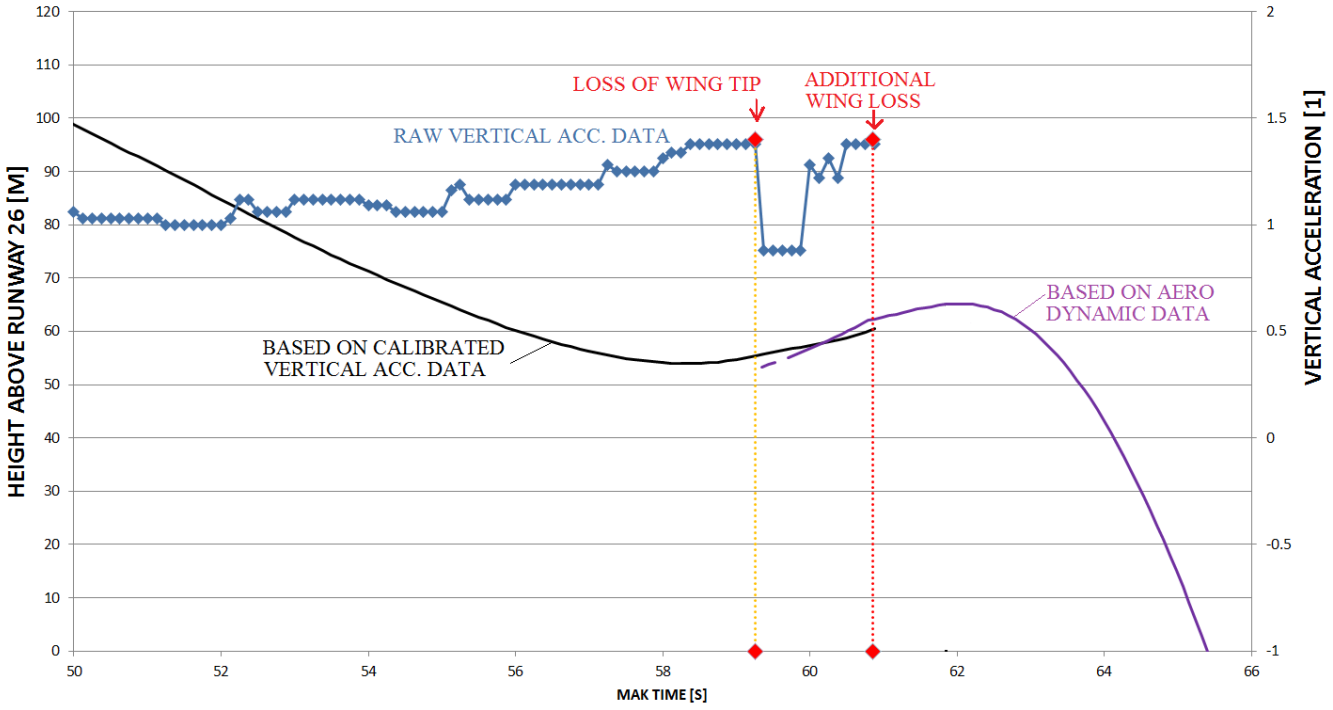

Fig. 7. The calculated trajectory based on <u>calibrated</u> vertical acceleration data (black line) agrees within a few meters in *X*, *Y* and *Z* with the trajectory based on aero dynamics (purple line) earlier presented. (Here the *Z* coordinate is shown).

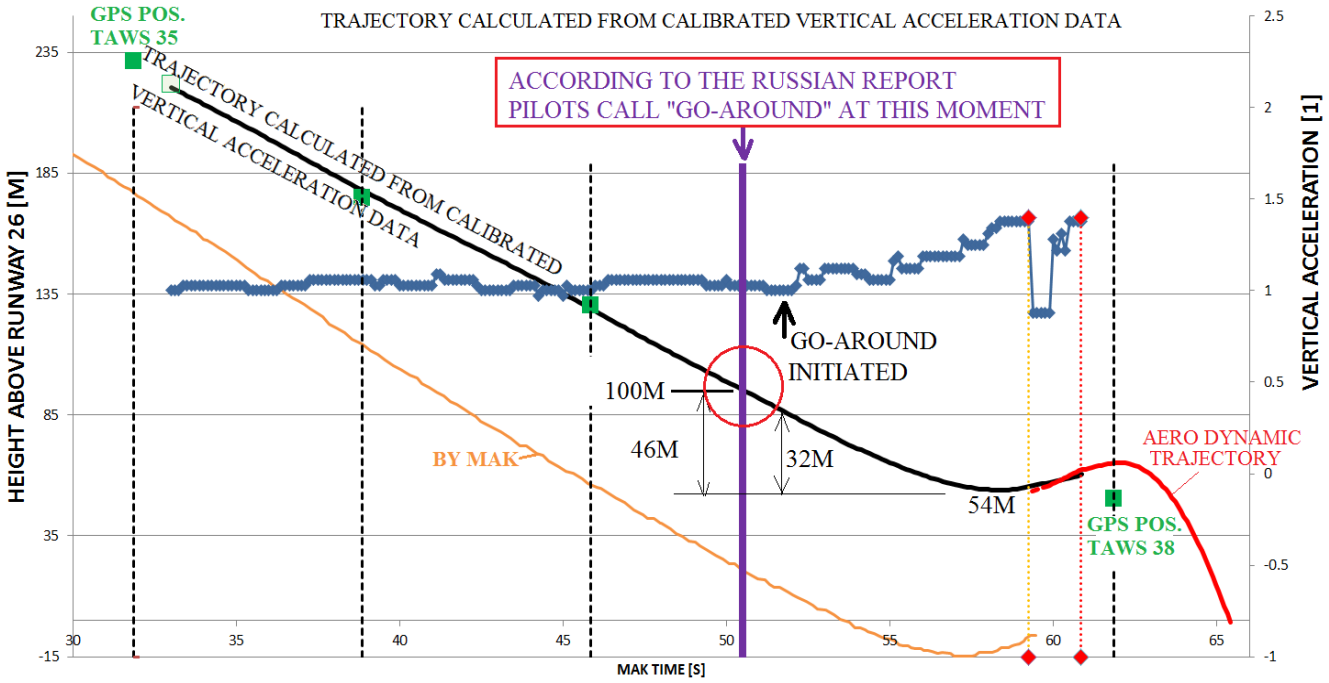

Fig. 8. The trajectory based on GPS black box data (black curve) and as earlier found by aerodynamic data (red curve).

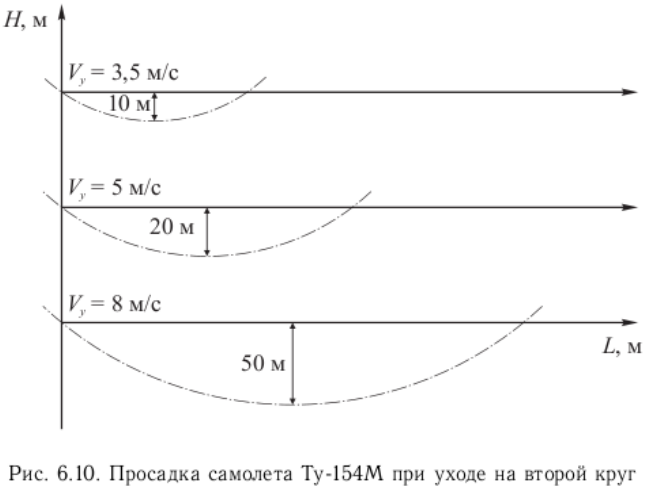

Рис. 6.10. Просадка самолета Ту-154М при уходе на второй круг

Fig. 9. The estimated height loss, ΔH, of the airplane during a go-around maneuver for three different values of vertical velocities *Vz* (-3.5 m/s, -5 m/s and -8 m/s). Based on this the height loss for an initial vertical velocity of *Vz* = -6.95 m/s is Δ*H* = 39.5 m [7].

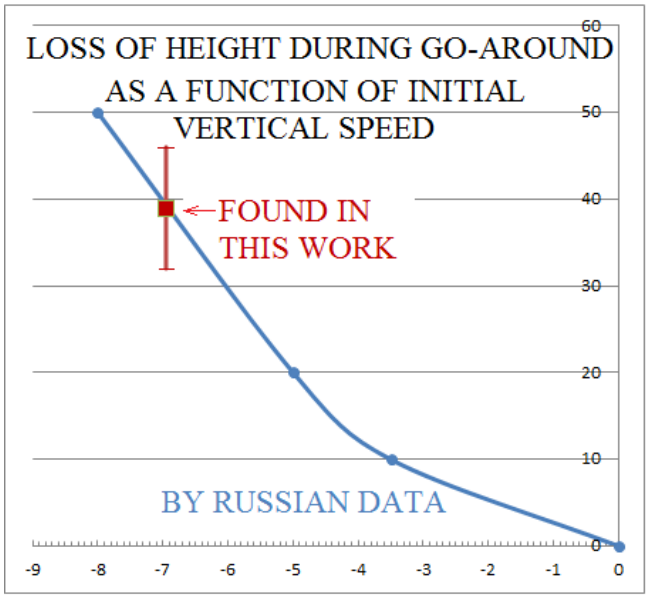

Fig. 10. The estimated height loss, Δ*H*, of the airplane during a go-around maneuver for three different values of vertical velocities *Vz* (-3.5 m/s, -5m/s and -8 m/s). Based on this the height loss for an initial vertical velocity of *Vz* = -6.95 m/s is Δ*H* = 39.5 m [6].

4. Conclusion

A robust method of calibrating the vertical acceleration sensor data by utilizing the recorded GPS data and the recorded vertical velocity data at the TAWS 35 to TAWS 37 events is found and demonstrated. The effect of uncertainties of the measured and recorded input parameters is evaluated using a Monte Carlo technique. The most likely height above the runway is found to be H = 57 m ± 9 m at the time the left wing tip was lost. This is in very good agreement with the independent trajectory found based on aero dynamics working from the crash site and up as earlier reported [6]. *With other words two independent analysis based on two very different sets of data and very different methods lead to the same result within a few meters.*

By the Monte Carlo simulations the plane was with 99.9 % certainty higher than 28 m above the ground in the vicinity of the birch tree claimed to have cut the wing, hereby invalidating the official explanation. The calculated vertical velocities agree very well with the recorded vertical velocities. The model can predict the vertical velocity at the Taws 38 event, and the small difference agrees with the

impact of the additional wing loss as found in [7], hereby confirming the model and the trajectory found. The calculated height loss of 32 m to 46 m is in good agreement with the 39.5 m for the TU-154M by the Russian litterature.

References

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